Introduction to the Inner Life of the Proton

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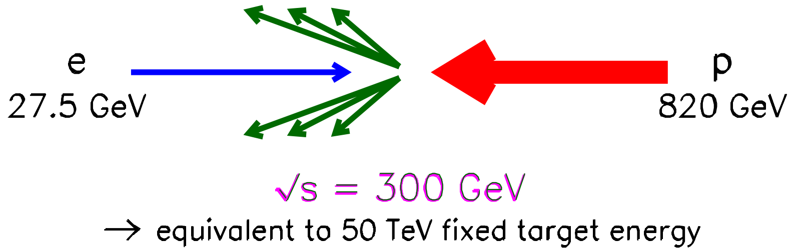
- The **H**adron **E**lectron **R**ing–**A**nlage
- **D**eep–**I**nelastic **S**cattering
- The **Q**uark **P**arton **M**odel
- **Q**uantum **C**hromo **D**ynamics
- The Strong Coupling Constant α_s
- **P**arton **D**ensity **F**unctions

Hadron Electron Ring–Anlage

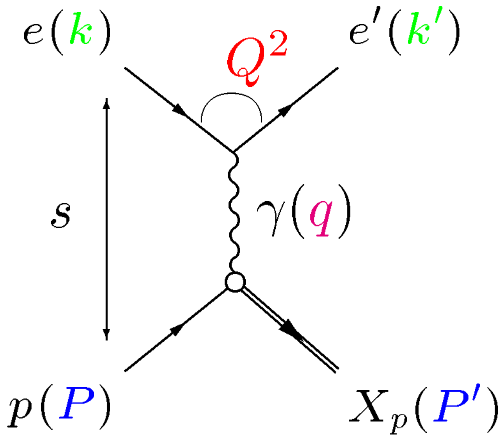
- Characteristics:
  - ep-collisions with $E_e = 27.5$ GeV and $E_p = 920$ GeV (in 1996/97: $E_p = 820$ GeV)
  - boosted final state



  - hadron in initial state

- Possibilities:
  - to understand $ep \to eX$ transition:
    precise determination of the partonic structure of the proton
  - test of theoretical QCD predictions
  - e.g. measurement of strong coupling constant α_s, diffractive processes etc.

HERA Kinematic Variables



negative four-momentum transfer squared:

$$Q^2 = -q^2 = -(k - k')^2$$

parton momentum fraction (Bjorken scaling variable):

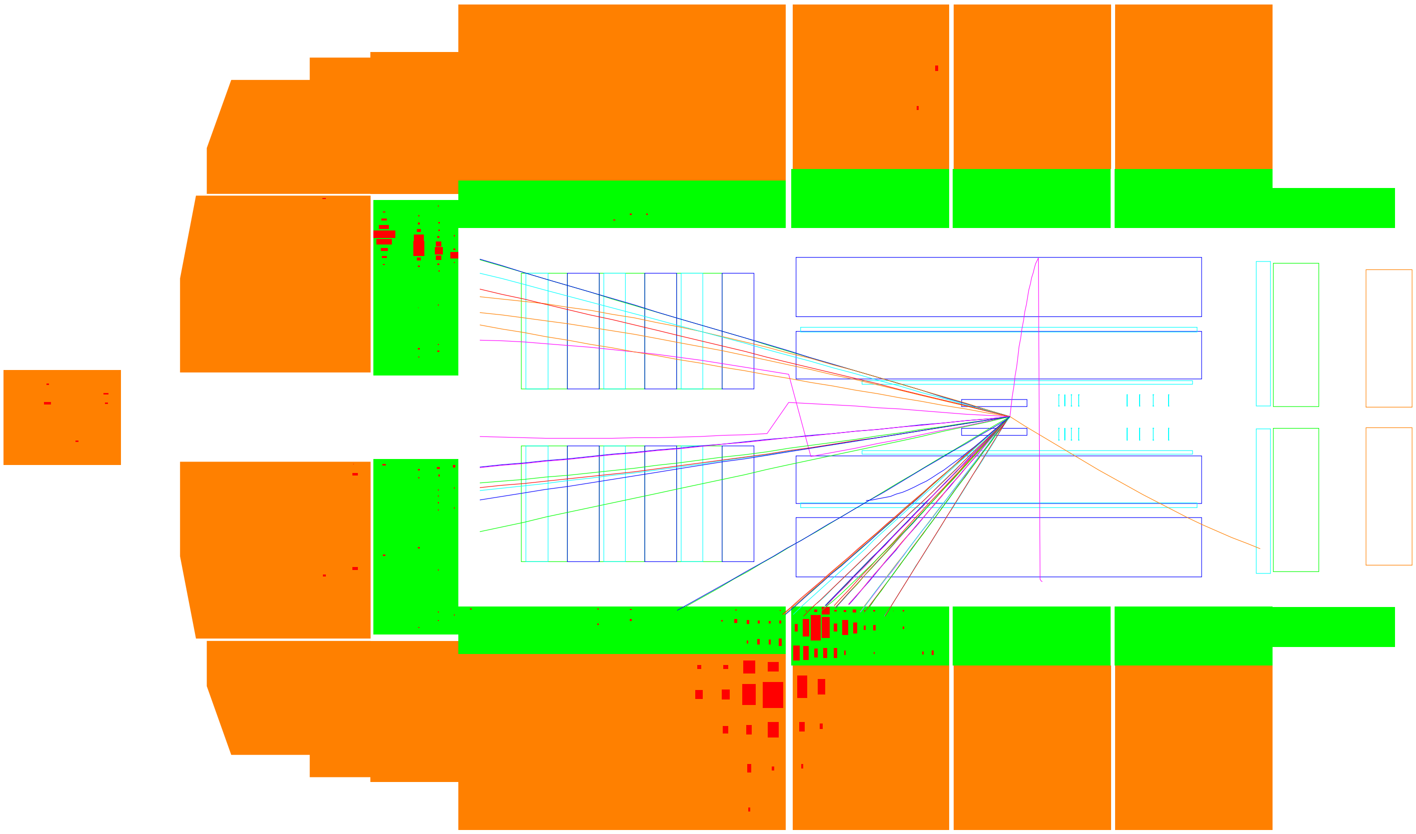
$$x = \frac{Q^2}{2P \cdot q}$$

inelasticity (fractional energy transfer in p rest frame):

$$y = \frac{q \cdot P}{k \cdot P} = 1 - \frac{E_{e'}}{E_e} \sin^2\left(\frac{\theta_e}{2}\right)$$

$$\Rightarrow Q^2 = x \cdot y \cdot s$$

The H1 Detector



$$Q^2 = 34000\ \text{GeV}^2$$
$$x = 0.46$$
$$y = 0.8$$

- $Q^2 \gg m^2_{Proton}$: deep-inelastic

- differential cross-section $ep \to eX$ (neglecting Z^0-exchange):

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{xQ^4} \cdot Y_+ \cdot [F_2(x, Q^2) - \frac{y^2}{Y_+} \cdot F_L(x, Q^2)]$$

- $\frac{y^2}{Y_+} = \frac{y^2}{1+(1-y)^2} < 1$ for all $y < 1$

- F_2 and F_L are structure functions, F_L corresponds to coupling of longitudinally polarized photons to the proton

- What are structure functions?

  - parameterization of the partonic structure of the proton
    $\Rightarrow$ Measurement of structure functions provides information about the inner life of the proton

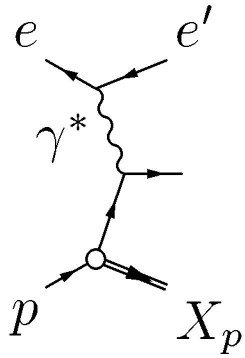
The Structure Function F_2

For non-interacting partons (Quark-Parton Model):
F_2 in terms of quark distributions $q_i(x)$:

$$F_2(x) = x \sum_i e_i^2 \cdot q_i(x)$$

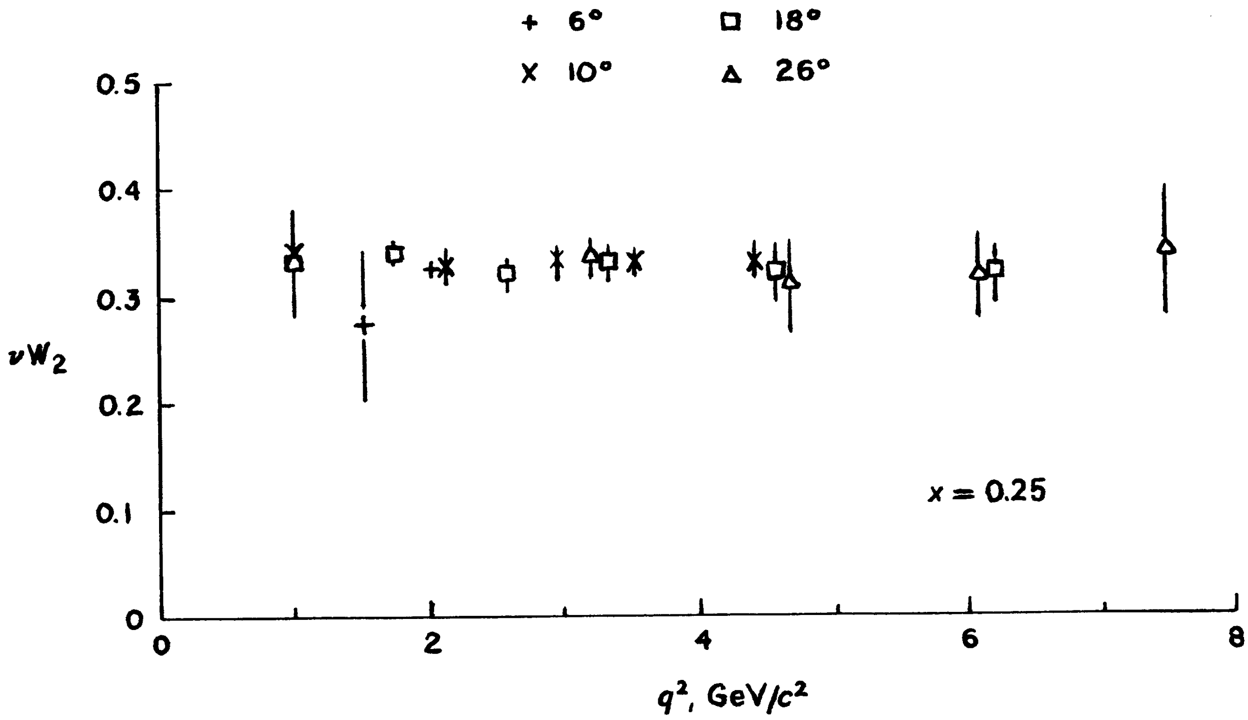
The Quark Parton Model

- virtual photon γ^* scatters off partons in the proton



- quarks are massless spin-$\frac{1}{2}$ particles without transverse momentum $k_\perp$
  $\Rightarrow$ no coupling of longitudinally polarized γ^* (due to spin constraints)
  $\Rightarrow$ $F_L = 0$

- In the QPM $F_2(x)$ does not depend on Q^2
  $\Rightarrow$ BJORKEN SCALING



E.D. Bloom et al. Phys.Rev.Lett. (1969) 930

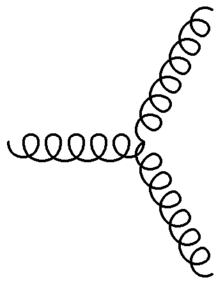
QuantumChromoDynamics

- QCD describes the strong interaction between quarks and gluons

- mathematical structure: non-abelian gauge invariant field theory

- gauge bosons in QCD are gluons

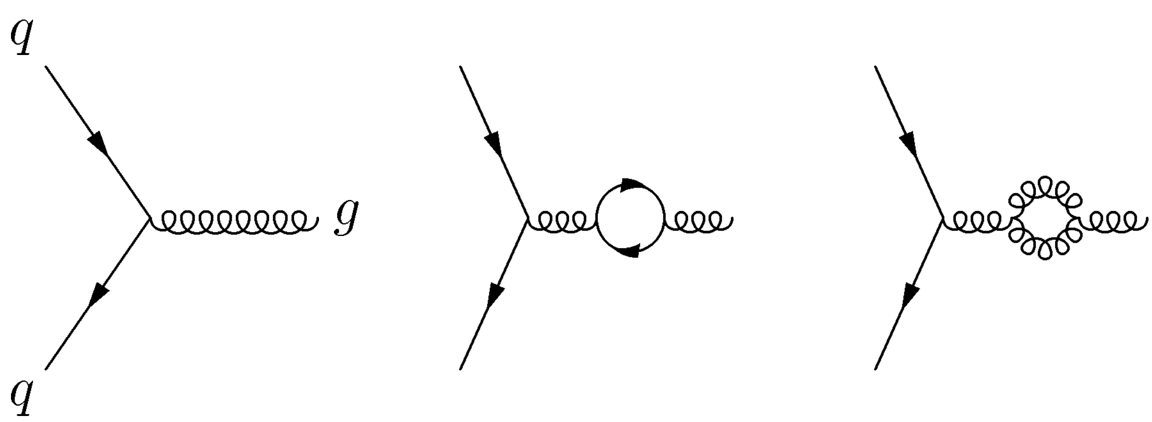
- quarks and gluons carry colour ($\leftrightarrow$ QED: photons have no charge)
  $\Rightarrow$ selfinteracting gluons:



- quarks can have transverse momentum due to gluon emission and absorption
  $\Rightarrow$ coupling to longitudinally polarized γ^* possible
  $\Rightarrow$ $F_L \neq 0$

- only free parameter of QCD:
  strong coupling constant α_s

- QCD predictions for cross-section using Feynman rules:



- virtual loop corrections ⇒ infinite integrals
  ⇒ renormalization and regularization

- regularization of divergent integrals by defining upper integration boundary

- renormalization by redefinition of coupling constant for all other terms (introduction of renormalization scale μ_R and scheme i.e. $\overline{MS}$)

- requirement: observables independent of μ_R:
  ⇒ Renormalization Group Equation

⇒ consequence: $\boxed{\alpha_s := \alpha_s(\mu_R)}$

- Solution for α_s in leading order:

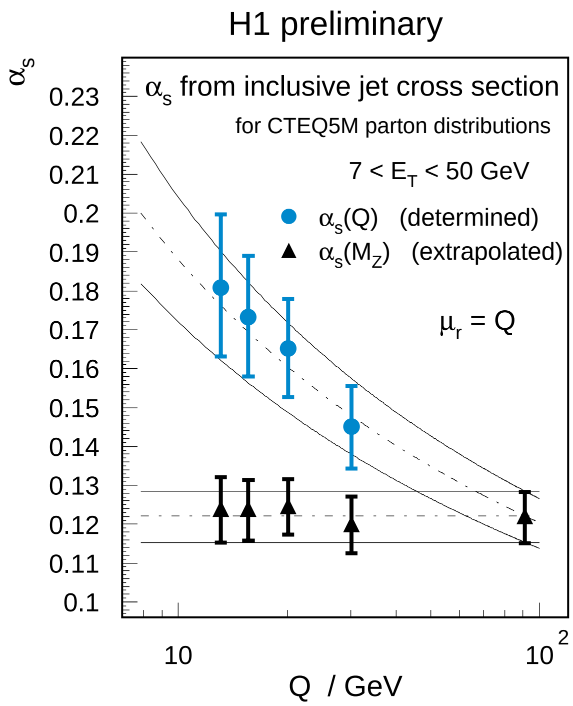
$$\alpha_s(Q^2) = \frac{\alpha_s(\mu_R^2)}{1 + \frac{\alpha_s(\mu_R^2)}{12\pi}(33 - 2n_f)\ln(\frac{Q^2}{\mu_R^2})}$$

n_f = number of quark flavours

$\Rightarrow$ Running coupling

- $Q^2 < \mu_R^2 \Rightarrow \alpha_s(Q^2) > \alpha_s(\mu_R^2)$
  $Q^2 > \mu_R^2 \Rightarrow \alpha_s(Q^2) < \alpha_s(\mu_R^2)$

- standard measurement: $\alpha_s(M_Z)$ as reference scale

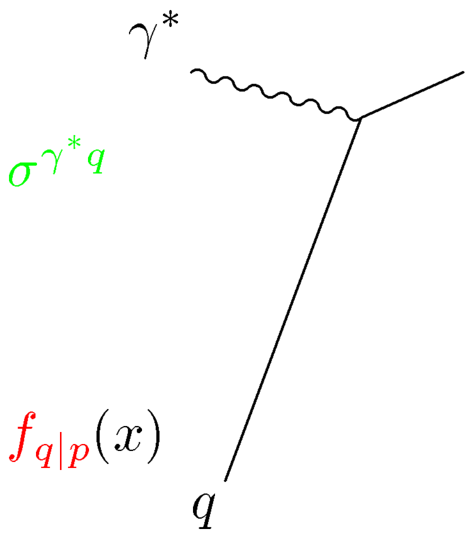


Asymptotic Freedom and Confinement

- short distance scale ($Q^2 \gtrsim$ 10 GeV) :
  γ^* interacts with partons
  $\Rightarrow$ asymptotic freedom
  $\Rightarrow$ calculable in perturbative QCD

- long distance scale ($Q^2 \lesssim 0.1$ GeV):
  partons appear in bound states
  $\Rightarrow$ confinement
  $\Rightarrow$ perturbative QCD not applicable

Deep-inelastic $\gamma^* p$ scattering =
incoherent sum of elastic $\gamma^* q$ scatterings



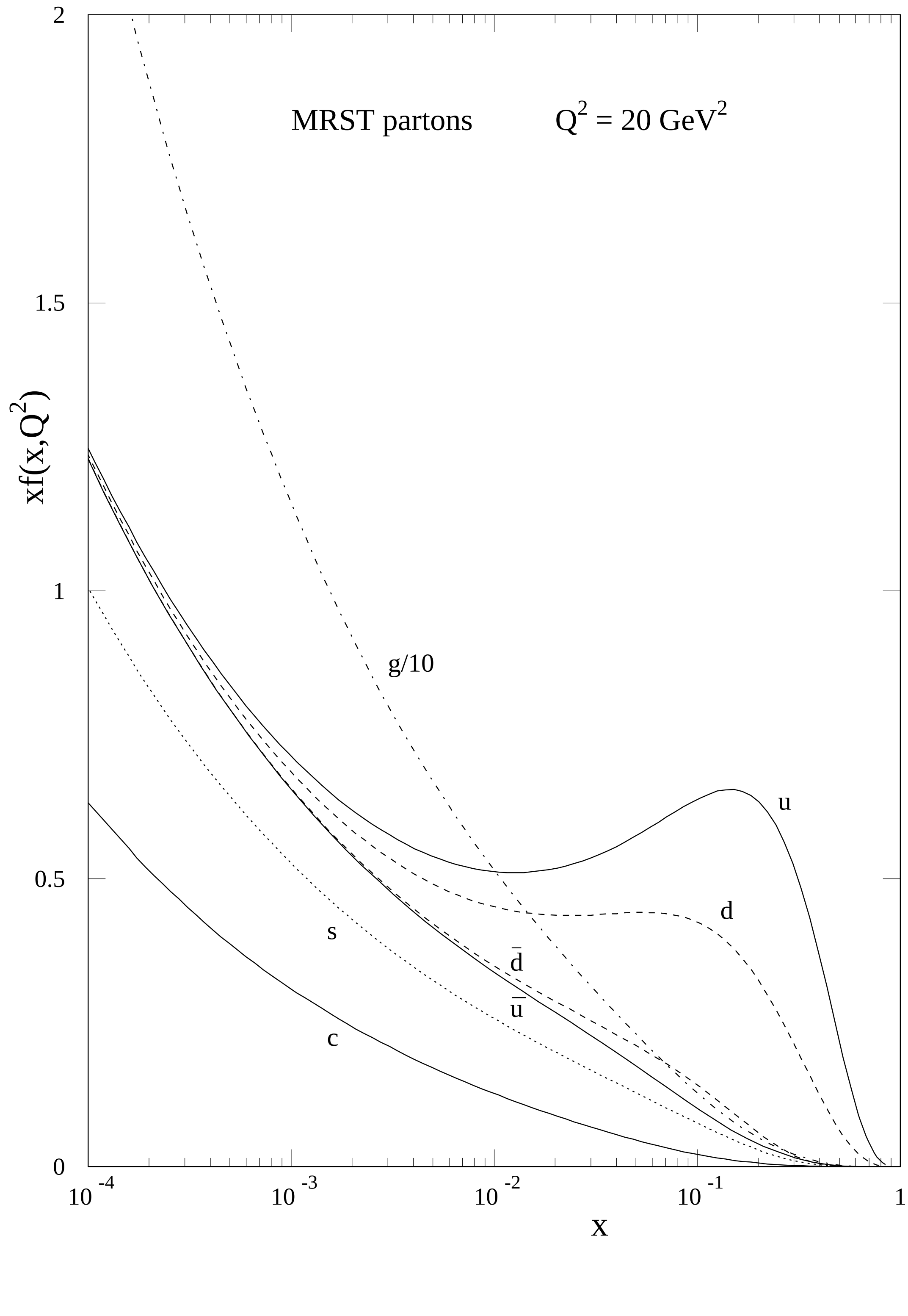
$\sigma^{\gamma^* q}$ =electron-quark cross-section

$f_{q|p}(x) \equiv q(x)$ = Parton density function (PDF)

Probability density to find quark q in the proton carrying a fraction x of the proton momentum

$$\sigma^{\gamma^* p} = \sum_{q,\bar{q}} \sigma^{\gamma^* q} \cdot f_{q|p}(x)$$

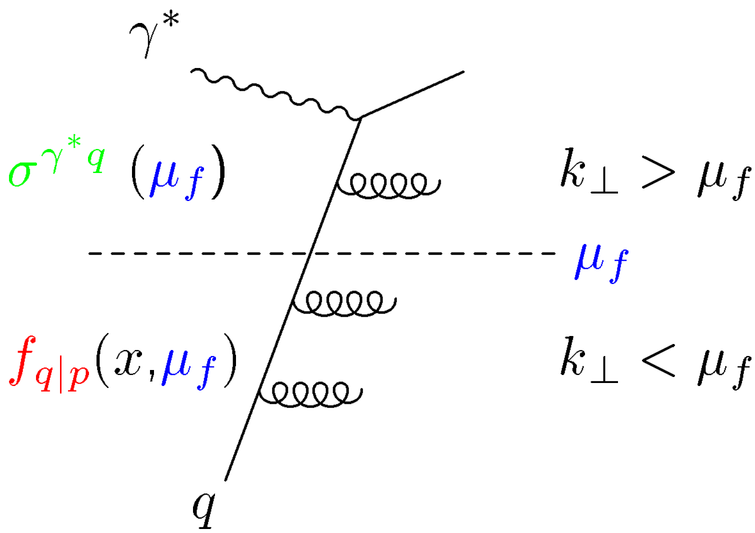
Factorization of perturbative and non-perturbative process



A.D. Martin et al. Eur.Phys.J. **C4**(1998) 463

- PDFs specify a universal property of the proton
(→ process independent)

- Higher order corrections:



- integration over all transverse momenta $k_\perp$
$\Rightarrow$ divergences for $k_\perp \to 0$ (infrared divergences)

- curing divergences: redefinition of PDFs (introduction of factorization scale μ_f and scheme i.e. $\overline{MS}$)

- for $k_\perp < \mu_f$: parton emissions are included in PDFs $f_{q|p}$

$$\sigma^{\gamma^* p} = \sum_{q,\bar{q}} \sigma^{\gamma^* q}(\mu_f^2) \cdot f_{q|p}(\mu_f^2)$$

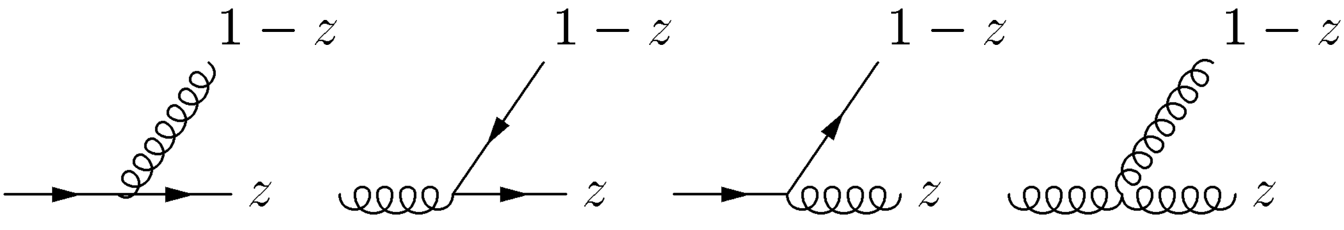
DokshitzerGribovLipatovAltarelliParisi–equation

- analogous to renormalization: no dependency of observables on μ_f is required
  $\Rightarrow$ analogue to renormalization group equation:

  Dokshitzer–Gribov–Lipatov–Altarelli–Parisi–equation

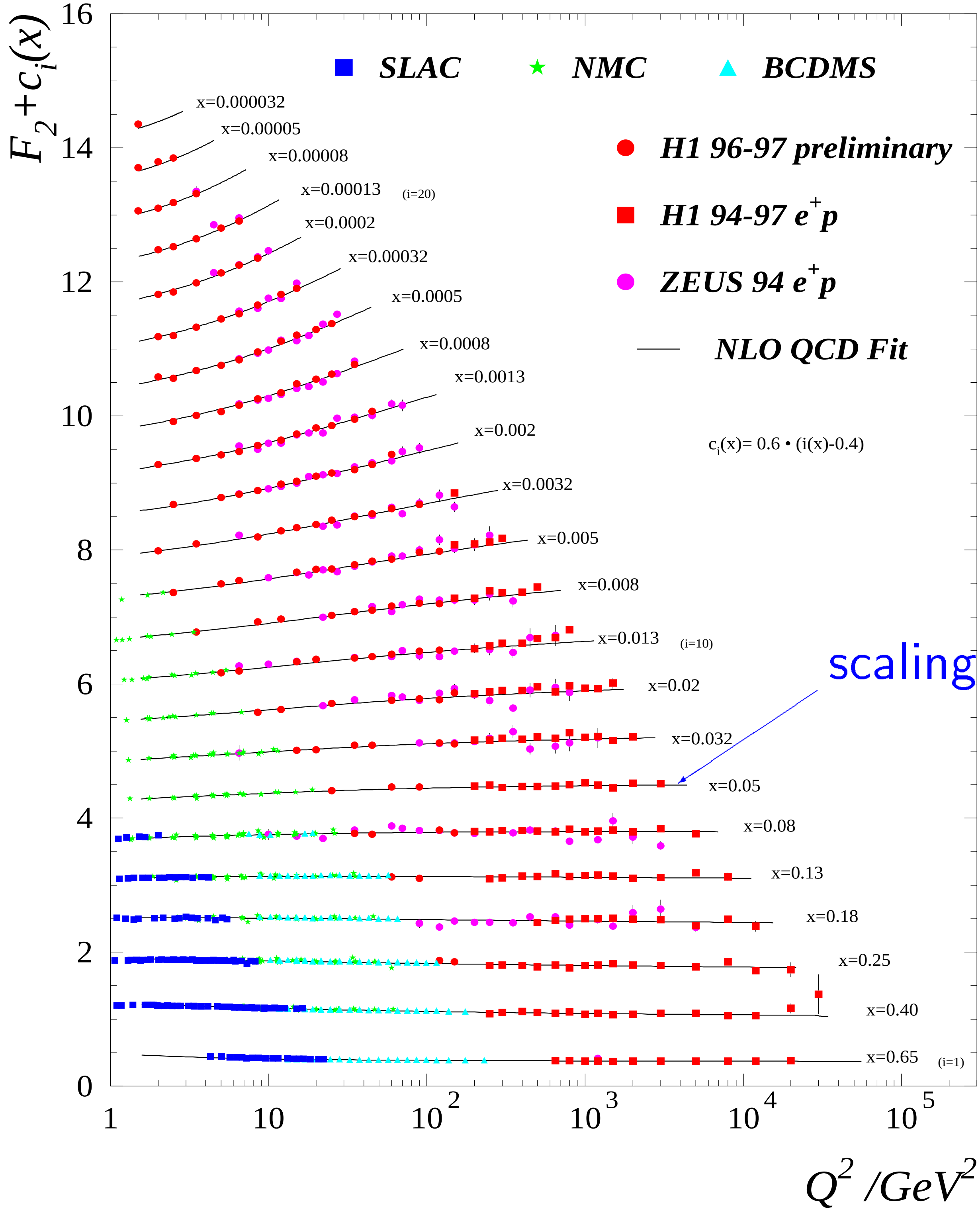
- system of coupled differential equations describing the μ_f dependence of the PDFs

- they include splitting functions ${P_{ab}}^0(z)$: probability density of finding parton a in parton b:

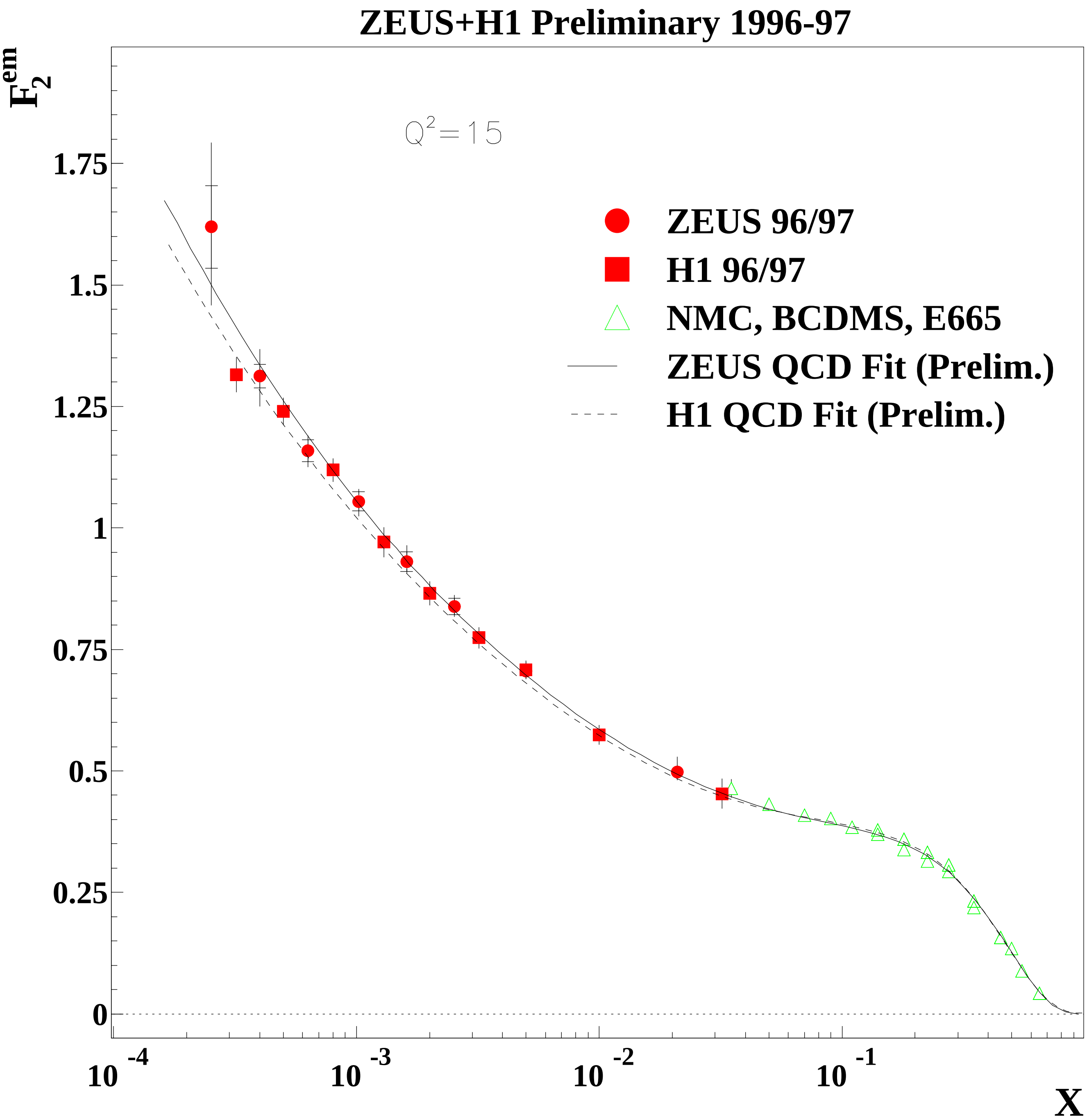


${P_{qq}}^0(z)$ ${P_{qg}}^0(z)$ ${P_{gq}}^0(z)$ ${P_{gg}}^0(z)$

$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 dz\left[P_{qq}(z)\cdot F_2\left(\frac{x}{z},Q^2\right)+ P_{qg}(z)\cdot 2n_f\cdot\frac{5}{18}\cdot\frac{x}{z}\cdot g\left(\frac{x}{z},Q^2\right)\right]$$



Overview of F_2



- fixed target experiments measured high x
  ⇒ valence quarks
- strong rise of F_2 with decreasing x
  ⇒ sea quarks

Summary

- DIS processes at HERA provide information about the partonic structure of the proton

- Quark Parton Model: no parton-parton interaction in the proton $\rightarrow$ Bjorken Scaling

- QCD: parton interactions lead to logarithmic scaling violation mathematically described by DGLAP-equations

- α_s is a running coupling constant