# Measuring the Polarization Vector of the Positrons from Polarized Muon Decays

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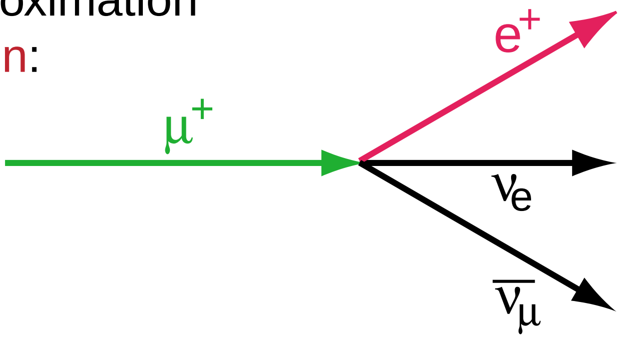
# Contents

- Motivation and Theoretical Background
- the Experiment at PSI
- Principles of Measuring the ...
  - ... Transverse Polarization Components
  - ... Longitudinal Polarization
- Results up to now
- Discussion of Methods for Further Data-Analysis
- Conclusion and Outlook

# Theory of Muon-Decay

In the limit of small momentum transfer $m_W >> q^2$:

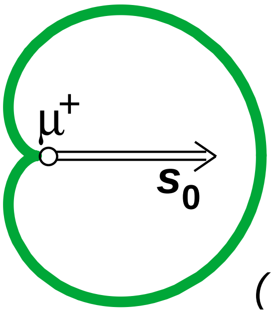
μ-decay described using approximation of four fermion point interaction:



## Decay Asymmetry:

polarized positive muons emit the decay positron preferably in the direction of the muon spin

$$\frac{d\Gamma}{d\cos\vartheta} \sim 1 + 1/3\, P_\mu \cos\vartheta$$



( for x = 1 )

$$x = \frac{E_{e^+}}{E_{max}}$$ is the reduced positron energy

$$E_{max} = \frac{m_e^2 + m_\mu^2}{2\, m_\mu}$$

$$x_0 \le x \le 1$$

$$x_0 = \frac{m_e}{E_{max}}$$

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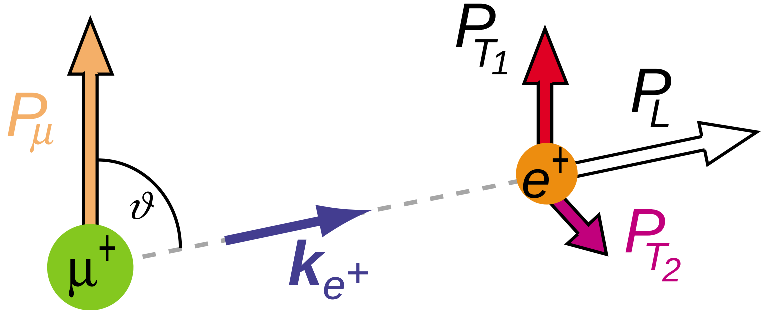
# Obseravbles in the Decay of Polarized Muons: Polarization Components of the Positrons

$$P_{\mathrm{T}_1} = f_1(E, \vartheta, \eta, \eta'') \,,$$

$$P_{\mathrm{T}_2} = f_2(E, \vartheta, \tfrac{\alpha'}{A}, \tfrac{\beta'}{A}) \,,$$

where $\eta, \eta'', \frac{\alpha'}{A}$ and $\frac{\beta'}{A}$ are Michel Parameters.



For $\vartheta = \pi/2$:

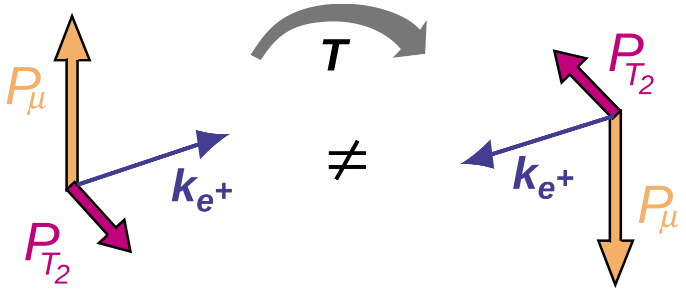
$$P_{T_1}(x) = \frac{x_0(1-x)}{2x^2 - 3x + x_0^2} - \frac{1}{2} \cdot \frac{\eta \cdot 3x + \eta''(3x-4)}{3-2x}$$

$$P_{T_2}(x) = 0 + 2\,\frac{3\frac{\alpha'}{A}(1-x) + 2\frac{\beta'}{A}}{3-2x}$$

SM predictions: $\eta = \eta'' = \frac{\alpha'}{A} = \frac{\beta'}{A} = 0$

$$\Rightarrow \langle P_{\mathrm{T}_1} \rangle_E = 0.003 \,, \; P_{\mathrm{T}_2} \equiv 0$$

A non-zero $P_{\mathrm{T}_2}$ violates time reversal invariance:



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# Matrix element for Muon Decay

$$\mathcal{M} = \frac{4G_\mathrm{F}}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \varepsilon,\mu=R,L}} g^{\gamma}_{\varepsilon\mu} \langle \overline{e}_\varepsilon | \Gamma^\gamma | (\nu_e)_n \rangle \langle \overline{\nu}_m | \Gamma_\gamma | (\mu)_\mu \rangle$$

$\gamma$ labels the type of interaction:

$$\Gamma^S = \text{4-scalar}$$
$$\Gamma^V = \text{4-vector}$$
$$\Gamma^T = \text{4-tensor}$$

$\varepsilon, \mu$ indicate the chiralities of the spinors of the observed (charged) leptons.

$n, m$ indicate the chiralities of the neutrinos, which are uniquely determined for given $\gamma, \varepsilon$ and $\mu$.

Measuring $P_{\mathrm{T}_1}$ yields the low energy parameter $\eta$ *without* the suppression factor $m_e/m_\mu$:

$$\eta = \frac{1}{2} Re \left\{ g^V_{LL} g^{S*}_{RR} + g^V_{RR} g^{S*}_{LL} + g^V_{LR} (g^{S*}_{RL} + g^{T*}_{RL}) + g^V_{RL} (g^{S*}_{LR} + g^{T*}_{LR}) \right\}$$

In the Standard Model:

$$g^V_{LL} = 1$$
$$g^{\gamma}_{\varepsilon\mu} = 0 \quad \text{(all other interactions)}$$

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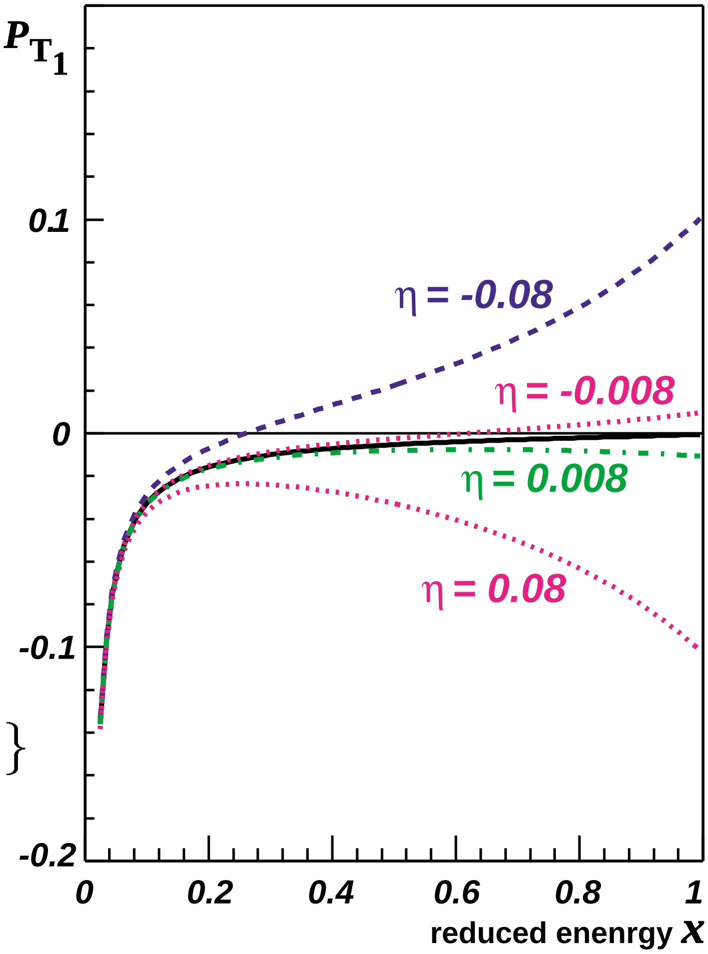
# Beyond the Standard Model

If one assumes that there is one additional coupling, a non-standard result for $P_\mathrm{T}$ would correspond to a non-zero $g^S_{RR}$.

Assuming $g^S_{RR} \neq 0$ and knowing that $g^V_{LL} \approx 1$, one deduces:



$$P_{\mathrm{T}_1}(E_e) \rightarrow \eta \approx \frac{1}{2} Re\{g^S_{RR}\}$$

$$P_{\mathrm{T}_2}(E_e) \rightarrow \frac{\beta'}{A} \approx \frac{1}{4} Im\{g^S_{RR}\}$$

| current limits | experimental aim |
|---|---|
| $\langle P_{\mathrm{T}_1} \rangle = 0.016 \pm 0.021$ ¹⁾ | $\Delta \langle P_{\mathrm{T}_1} \rangle = 0.003$ |
| $\langle P_{\mathrm{T}_2} \rangle = 0.007 \pm 0.022$ ¹⁾ | $\Delta \langle P_{\mathrm{T}_2} \rangle = 0.003$ |
| $\langle P_\mathrm{L} \rangle = 1.00 \pm 0.04$ ²⁾ | |

1): H. Burkhard et al., 1985 , 2): average for several experiments

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# Main Scientific Interests,
# Motivation for the Experiment at PSI

1. search for new couplings beyond V - A :

   assuming the presence of one addditional coupling, the coupling constant $g^S_{RR}$ can be determined from the Michel parameters $\eta$ and $\frac{\beta'}{A}$

2. violation of time reversal invariance :

   a non-zero $P_{T_2}$ violates time reversal invariance !

3. model independent determination of the Fermi coupling constant $G_F$ :

$$G_F = \sqrt{192\,\pi^3 \frac{1}{\tau_\mu\, m_\mu^5} \left(1 - 4\eta \frac{m_e}{m_\mu}\right)}$$

$$\textit{contributions to } \left(\frac{\Delta G_F}{G_F}\right)^2 : \quad \begin{array}{ll} \Delta\tau_\mu : & 8 \cdot 10^{-11} \\ \Delta m_\mu : & 6 \cdot 10^{-13} \\ \Delta\eta\ : & 1{,}6 \cdot 10^{-8} \end{array}$$

$\eta$ can be determined via measurement of $P_{T_1}$

→ measure $P_{T_1}$ and $P_{T_2}$

\+ measure $P_L$ as a check of consistency
and to complete the polarization vector

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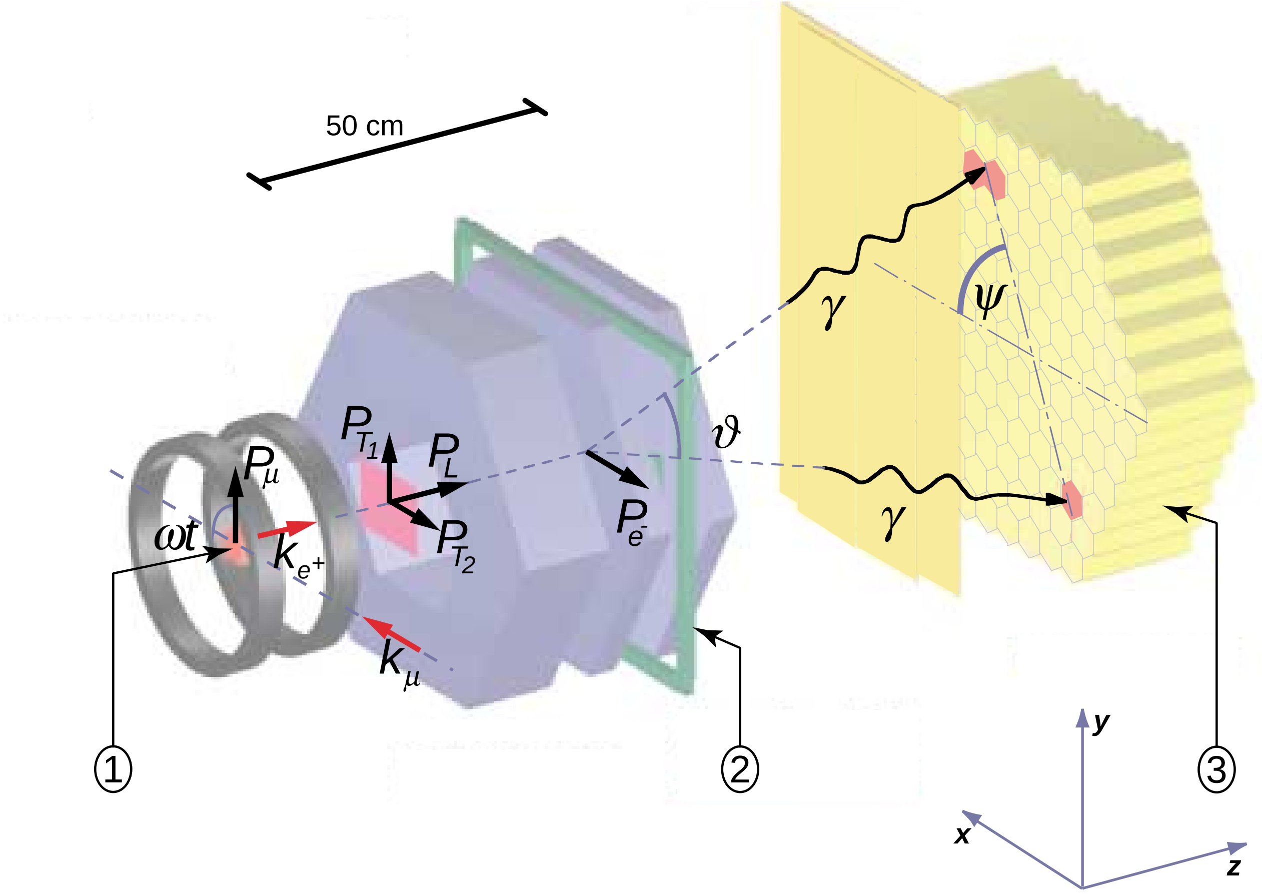
# Setup of the Experiment and Principle of Measurement



① : Beryllium stop target within spin precession magnet

② : magnetized Vacoflux foil within iron return yoke

③ : calorimeter consisting of 127 BGO crystals

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# Experimental Methods for measuring all 3 Components of the Positrons' Polarization Vector

$$\boldsymbol{P}_{e^+} = \begin{pmatrix} P_{\mathsf{T}_1} \\ P_{\mathsf{T}_2} \\ P_{\mathsf{L}} \end{pmatrix} \equiv \begin{pmatrix} P_{\mathsf{T}} \cdot \cos\varphi \\ P_{\mathsf{T}} \cdot \sin\varphi \\ P_{\mathsf{L}} \end{pmatrix}$$

with 3 simultaneous and independent measurements:

| Observable | Method |
|---|---|
| $P_{\mathsf{T}}$ | Time dependence of annihilation |
| $\varphi$ | Remnant $\mu$SR $^{1)}$ effect |
| $P_{\mathsf{L}}$ | Spatial dependence of annihilation |

1) : $\mu$SR = “Muon Spin Rotation”

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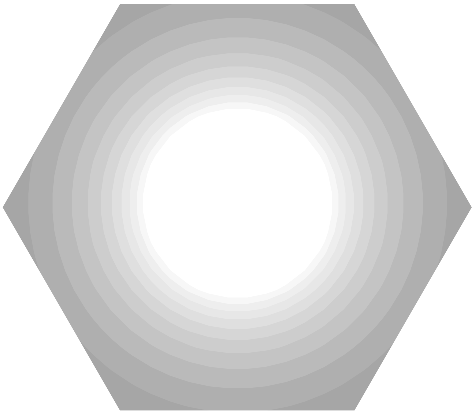
# Measurement of $|P_T|$

## *using the* Dependence *of the* Annihilation Cross-Section *on the* Relative Orientation of Spins

simulations of photon intensity
distributions on BGO - wall :

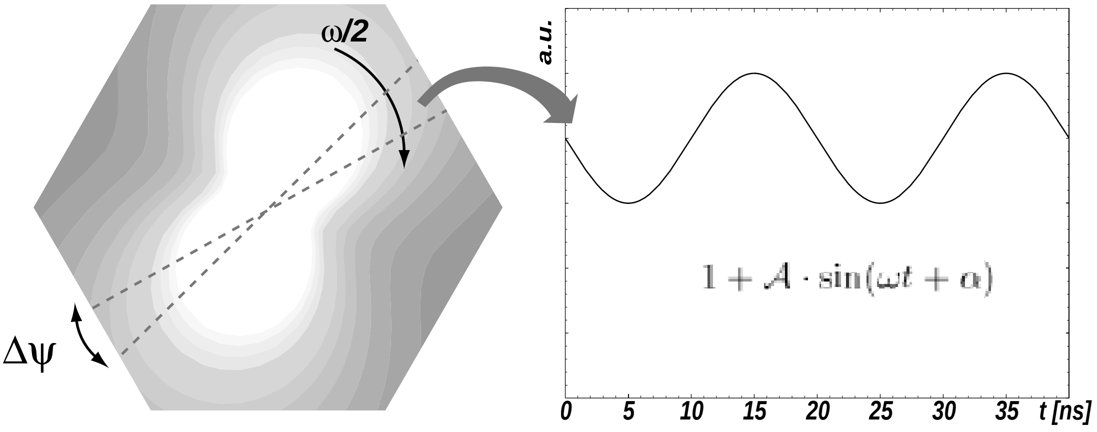


$|P_T| = 0$

***Assumptions:***

***$P_T$@ 45°, $E_{\gamma_1} = E_{\gamma_2} = 10$ MeV,***

***$e^-$-polarisation in magn. foil =100%***



$|P_T| = 1$

***time spectrum for a given $\Delta\Psi$***
***$A$ and $\alpha$ are functions of $P_{T_1}$ and $P_{T_2}$***

*$\omega$ : frequency of muon spin precession*
*$t$ : time between stop of muon and its decay*

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# Measurement of $|P_T|$

## Fitting *the* Time Dependence *of the* Signal observed in the BGOs

... using a Log Likelihood parameter estimation:

***Use the differential annihilation cross-section :***

$$\frac{1}{\sigma_0} \cdot \frac{d\sigma}{d\Omega} = 1 + \mathcal{A} \cdot \sin(\omega t + \alpha) \quad =: f(P_1, P_2, E_{\gamma_1}, E_{\gamma_2}, \Psi, t)$$

***where amplitude and phase are functions of the Energy and $\Psi$ :***

$$\mathcal{A} = \mathcal{A}(P_1, P_2, E_{\gamma_1}, E_{\gamma_2}, \Psi) \qquad \alpha = \alpha(P_1, P_2, E_{\gamma_1}, E_{\gamma_2}, \Psi)$$

***Likelihood Function :***

$$\begin{aligned} \mathcal{L}(P_1, P_2) &:= -\ln \prod_i^n f(P_1, P_2, E^i_{\gamma_1}, E^i_{\gamma_2}, \Psi^i, t^i) \\ &= -\sum_i^n \ln f^i(P_1, P_2) \end{aligned}$$

***where n is the number of "good" annihilation events***

→ determination of two perpendicular transverse polarization components $P_1$ , $P_2$ at the time of annihilation

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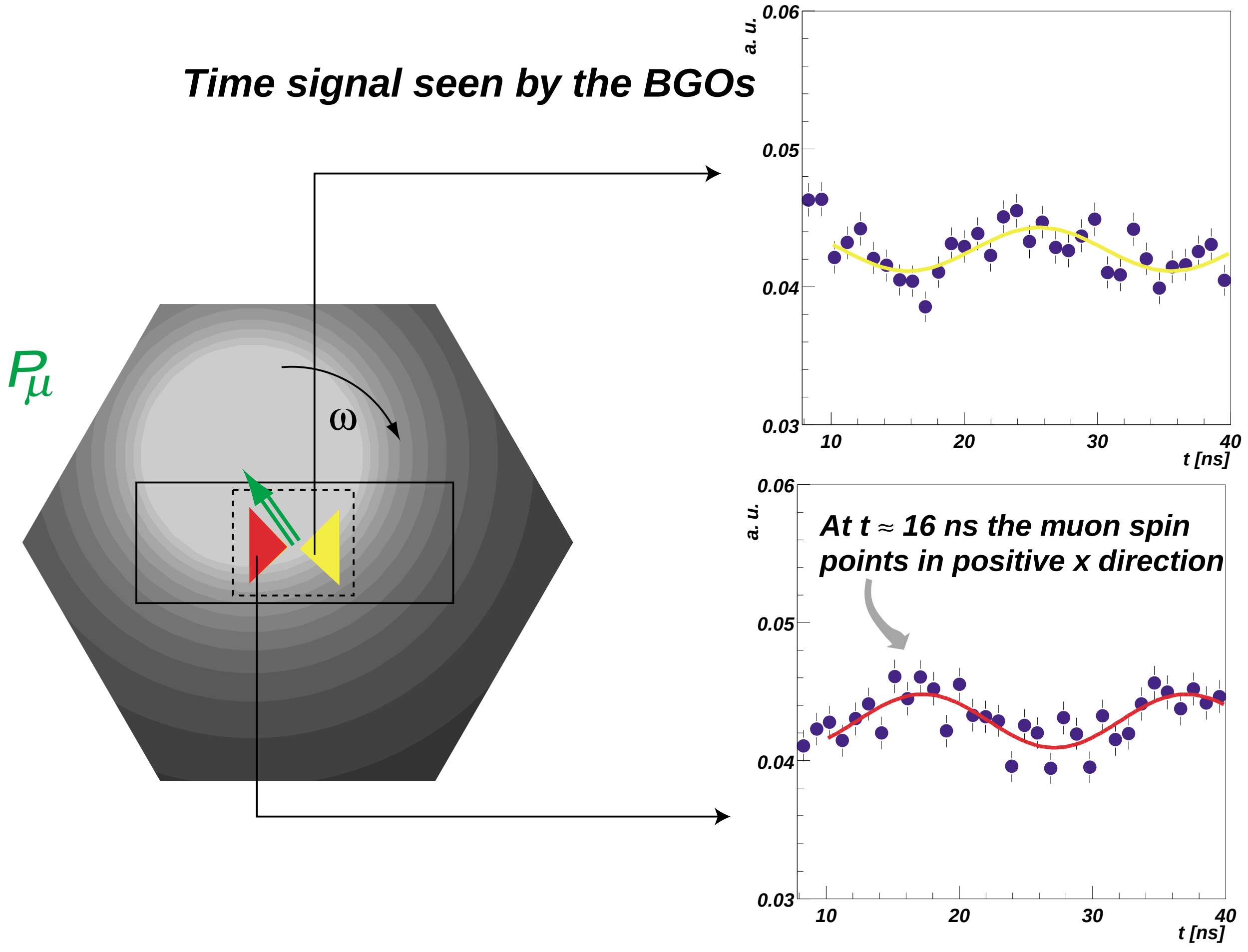
# Determination of the Phase φ of $P_T$

## *μSR Effect is used to find the direction of the muon spin*



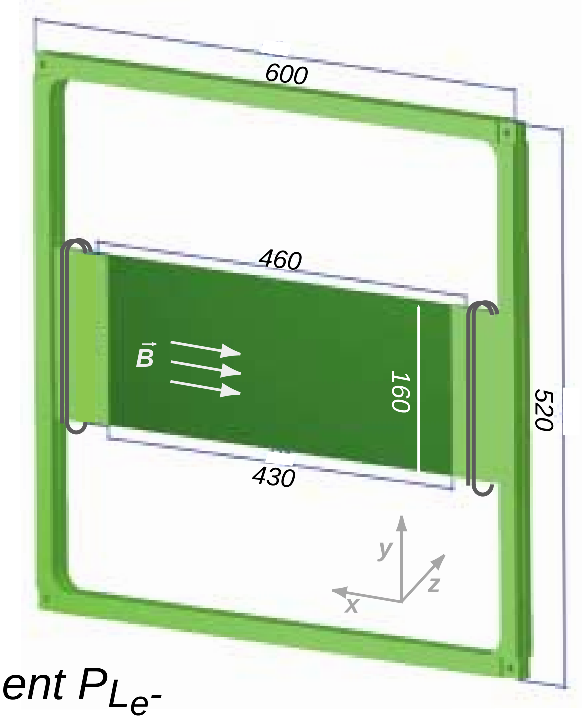
*Time signal seen by the BGOs*

*At t ≈ 16 ns the muon spin points in positive x direction*

results from 1999 data : $t_0 = 15.717 \pm 0.517$ ns

⟶ determination of "time zero", resp. the Phase φ of the transverse polarization

# Measurement of the Longitudinal Polarization



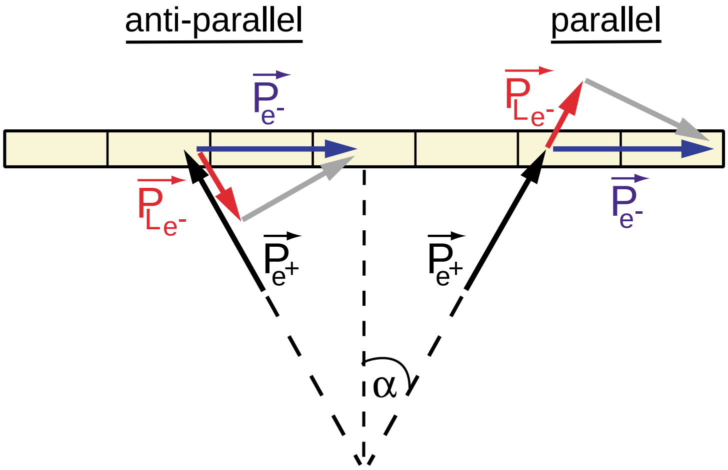
using information about position
on magnetized Vacoflux foil
(determined by tracks reconstructed
from drift-chamber data)
where annihilations take place

area on foil taken into account: $140^2$ mm$^2$

area divided into rectangular bins ( ij ),
17 bins in x- and y-direction, respectively

*Tracks that do not hit the center of the foil 'see' a longitudinal component $P_{Le^-}$ of the polarization of the electrons in the foil.*

*This $P_{Le^-}$ can either be parallel or anti-parallel to the positron polarization :*



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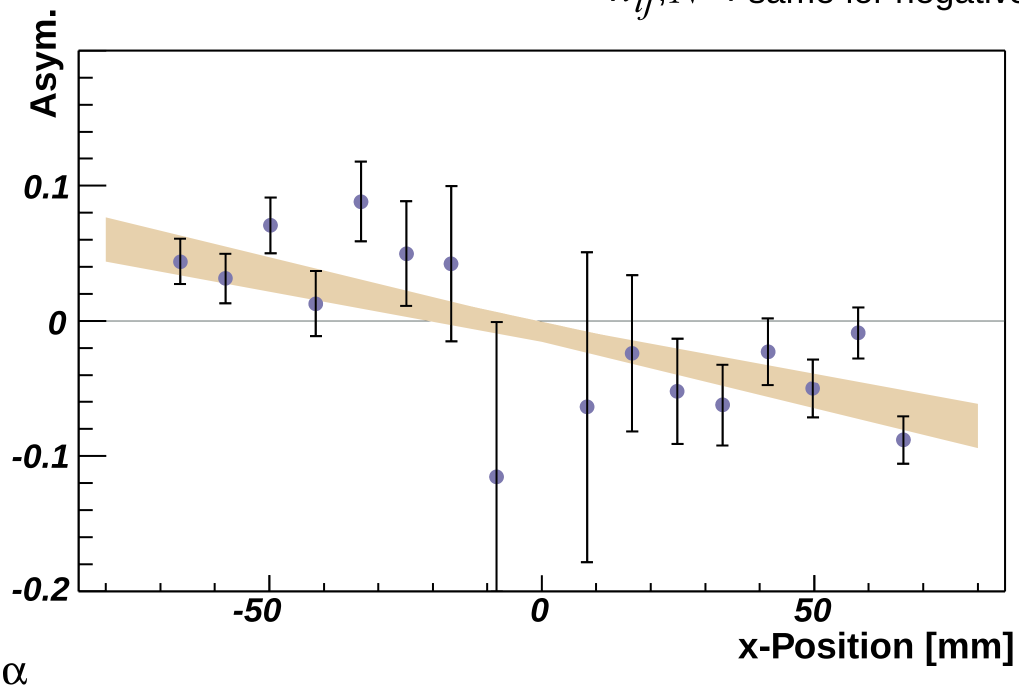
# Measurement of the Longitudinal Polarization

The annihilation cross section depends on the relative orientation of the spins of positrons and electrons in the foil. It is larger if both spins are anti-parallel.

Asymmetry: $A_{ij} = \dfrac{\dfrac{n_{ij}^{-}}{N^{-}} - \dfrac{n_{ij}^{+}}{N^{+}}}{\dfrac{n_{ij}^{-}}{N^{-}} + \dfrac{n_{ij}^{+}}{N^{+}}}$ where

$n_{ij}^{+}$ : number of annihilations in bin *ij* for positive foil polarization
$N^{+}$ : total number of annihilations for positive polarization
$n_{ij}^{-}, N^{-}$: same for negative polarization



- angle α
- elektron polarization in foil ($P_{e-}$ = 7.2 %)
- analysing power of 0.79
- background factor of 0.75
  (backgr. ratio 25 %, mainly due to bremsstrahlung)

$\langle P_{\mathrm{L}} \rangle_E$

# The following Results are based on the Data from the Datataking-Run in Fall 1999

## Rates [ $s^{-1}$ ] :

| | |
|---|---|
| $\mu^+$ stopped in target : | $20 \times 10^6$ |
| $e^+$ on magnetized foil : (startcounters) | $175 \times 10^3$ |
| startcounters but not vetocounters : | $37 \times 10^3$ |
| events triggered as annihilations : $( T \wedge A \wedge \Sigma BGO \wedge CRU )$ | 260 |

about 18 days of datataking ...

## Number of Events :

| | |
|---|---|
| total no. of raw events recorded as annihilations : | $240 \times 10^6$ |
| after all cuts :<br>( reconstructed track from target to foil, hit-info. of driftchambers to locate annihilation, kinematic consistency for annihilation events to exclude background ) | $27 \times 10^6$ |
| used events :<br>( technical problem with a TDC, exclusion of run periods with changes in setup parameters ) | $11 \times 10^6$ |

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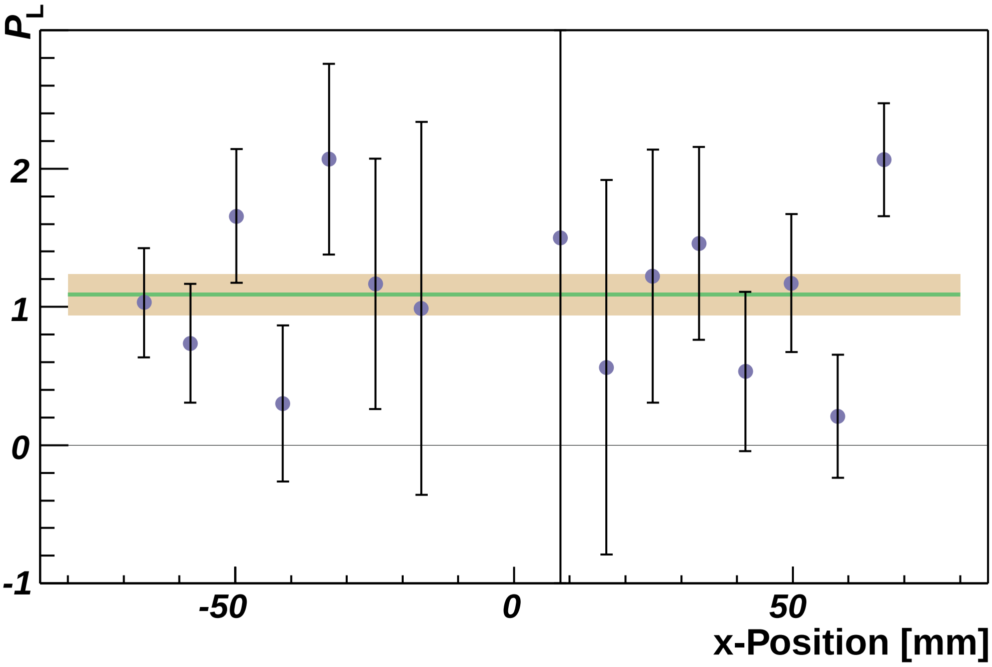
# Results: Longitudinal Polarization $P_L$

Asymmetry $A_{ij}$

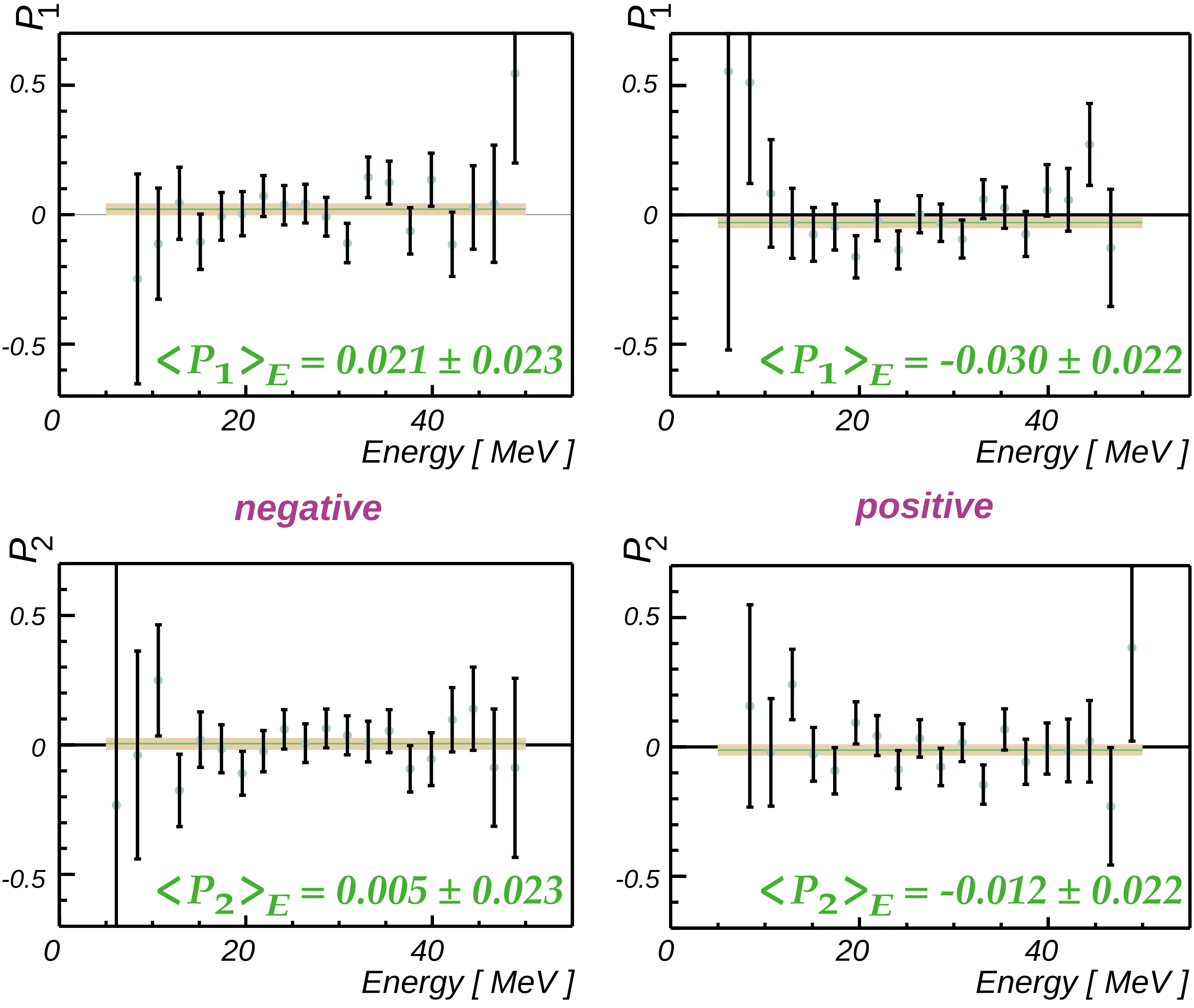
- angle $\alpha$
- elektron polarization $P_{e-}$
- analysing power
- background factor



$$\langle P_L \rangle_E = 1.09 \pm 0.15$$

# Results: $P_T$ at the Time of Annihilation

**Log Likelihood parameter estimation leads to the following results for the two different orientations of electron polarization in the magnetized foil:**



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# Next Steps in Data-Analysis:
# How to get from $P_1, P_2$ and $\varphi$ to $P_{\mathrm{T}_1}, P_{\mathrm{T}_2}$?

$P_{\mathrm{T}_1}$ and $P_{\mathrm{T}_2}$ are the transverse polarization components relative to $\boldsymbol{P}_\mu$ and $\boldsymbol{k}_{\mathrm{e}+}$ at the time of muon decay.

No analytical way to calculate $P_{\mathrm{T}_1}$ and $P_{\mathrm{T}_2}$!

⇒ using Monte-Carlo-Simulations:

The differential decay probability for an $\mathrm{e}^+$ with reduced energy between $x$ and $x + dx$, emitted at an angle of $\vartheta = \pi/2$ with respect to $\boldsymbol{P}_\mu$ is

$$\frac{d^2\Gamma}{dx} \sim G_F \cdot \sqrt{x_2 - x_0^2} \cdot F_{IS}(x) \cdot (1 + \boldsymbol{P}_{\mathrm{e}} \cdot \hat{\zeta}) \, ,$$

where $\boldsymbol{P}_{\mathrm{e}} = P_{\mathrm{T}_1} \cdot \hat{x_n} + P_{\mathrm{T}_2} \cdot \hat{y_n} + P_{\mathrm{L}} \cdot \hat{z}$ and $\hat{x_n}, \hat{y_n}, \hat{z_n}$ are the base vectors of the positron's individual coordinate system after $n$ interactions.

Neglecting $x_0$ (electron mass), the transverse polarization components become

$$P_{T_1}(x) \approx \frac{\eta \cdot 3x + \eta''(3x - 4)}{6 - 4x}$$

$$P_{T_2}(x) \approx \frac{\frac{3}{4}\frac{\alpha'}{A}(1 - x) + \frac{1}{2}\frac{\beta'}{A}}{6 - 4x}$$

These distributions are linear in $\eta, \eta'', \frac{\alpha'}{A}$ and $\frac{\beta'}{A}$ !

# Next Steps in Data-Analysis

We generate positrons having polarization distributions $P_i(x)\,, i = 0..4$ in Monte-Carlo.

For each $i \neq 0$ only one of the four Michel Parameters is $\neq 0$. For $i = 0$ the positrons are unpolarized.

⇒ The tracking of the positrons and dealing with their polarization in the Monte-Carlo-simulations leads to distributions $V_i(x)$ for the polarization at the time of annihilation in the magnetized foil.

⟹ The distribution $V(x)$ as <u>measured</u> is fitted using the $V_i(x)$ :

$$V(x) = \sum_{i=0}^{4} \alpha_i \, V_i$$

The resulting values for the $\alpha_i$ yield the values for the Michel Parameters as determined by means of our data-taking.

By correctly describing the energy-loss of positrons reaching the BGOs ("Michel Positrons") our Monte-Carlo-program will prove its trustability.

... work is in progress here ...

... after some tests were are still optimistic ...

# Conclusion and Outlook

- Successful Measurement of $P_L$ demonstrates that the apparatus is sensitive to polarization effects
- Determination of the Transverse Polarization $P_T$ at the time of annihilation based on the data taken in 1999; statistical errors in the range of the previous exp. limits
- A Method to deduce the actual components $P_{T_1}$ and $P_{T_2}$ is beeing developed

→ **First measurement of all three positron polarization components**

→ **New limits on Michel parameters $\eta$, $\eta''$, $\alpha'/A$, $\beta'/A$, new limits on additional couplings will be set**

→ **New measurement of the model independent $G_F$ will be possible**

***up to now, the transverse polarization is within the errors compatible with zero → no hints for physics beyond SM***

- More Data was taken in Nov. 2000 :

**29 days of datataking,**

**≈ 3 times higher event rate than in 1999**

**$1.37 \times 10^9$ raw annihilation events recorded,**

**≈ 13 times more "good" annihilations than in 1999**

→ ***reduction of $\Delta < P_T >$ to the aspired value of $\pm$ 0.003 should be possible***

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